Paper Reference(s)

6663/01

Edexcel GCE

Core Mathematics C1

Advanced Subsidiary

Surds and Indices

Calculators may NOT be used for these questions.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' might be needed for some questions.

The marks for the parts of questions are shown in round brackets, e.g. **(2)**.

There are xx questions in this test.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.

You must show sufficient working to make your methods clear.

Answers without working may not gain full credit.

1. Write

$$\sqrt{(75)} - \sqrt{(27)}$$

in the form $k\sqrt{x}$, where k and x are integers.

(Total 2 marks)

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2. (a) Expand and simplify $(7 + \sqrt{5})(3 - \sqrt{5})$.

(3)

(b) Express $\dfrac{7+\sqrt{5}}{3+\sqrt{5}}$ in the form $a + b\sqrt{5}$, where a and b are integers.

(3)
(Total 6 marks)

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3. Simplify

(a) $(3\sqrt{7})^2$

(1)

(b) $(8 + \sqrt{5})(2 - \sqrt{5})$

(3)
(Total 4 marks)

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4. Given that $32\sqrt{2} = 2^a$, find the value of a.

(Total 3 marks)

5.
$$f(x) = \frac{(3 - 4\sqrt{x})^2}{\sqrt{x}}, \ x > 0$$

(a) Show that $f(x) = 9x^{-\frac{1}{2}} + Ax^{\frac{1}{2}} + B$, where A and B are constants to be found.

(3)

(b) Find $f'(x)$.

(3)

(c) Evaluate $f'(9)$.

(2)
(Total 8 marks)

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6. (a) Write down the value of $125^{\frac{1}{3}}$.

(1)

(b) Find the value of $125^{-\frac{2}{3}}$.

(2)
(Total 3 marks)

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7. Expand and simplify $(\sqrt{7} + 2)(\sqrt{7} - 2)$.

(Total 2 marks)

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8. Given that $\dfrac{2x^2 - x^{\frac{3}{2}}}{\sqrt{x}}$ can be written in the form $2x^p - x^q$,

(a) write down the value of p and the value of q.

(2)

Given that $y = 5x^2 - 3 + \dfrac{2x^2 - x^{\frac{3}{2}}}{\sqrt{x}}$,

(b) find $\dfrac{dy}{dx}$, simplifying the coefficient of each term.

(4)
(Total 6 marks)

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9. (a) Write down the value of $16^{\frac{1}{4}}$.

(1)

(b) Simplify $(16x^{12})^{\frac{3}{4}}$.

(2)
(Total 3 marks)

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10. Simplify

$$\frac{5 - \sqrt{3}}{2 + \sqrt{3}},$$

giving your answer in the form $a + b\sqrt{3}$, where a and b are integers.

(Total 4 marks)

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11. Simplify $(3 + \sqrt{5})(3 - \sqrt{5})$.

(Total 2 marks)

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12. (a) Find the value of $8^{\frac{4}{3}}$.

(2)

(b) Simplify $\frac{15x^{\frac{4}{3}}}{3x}$.

(2)
(Total 4 marks)

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13. (a) Express $\sqrt{108}$ in the form $a\sqrt{3}$, where a is an integer.

(1)

(b) Express $(2 - \sqrt{3})^2$ in the form $b + c\sqrt{3}$, where b and c are integers to be found.

(3)
(Total 4 marks)

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14. (a) Expand and simplify $(4 + \sqrt{3})(4 - \sqrt{3})$.

(2)

(b) Express $\frac{26}{4+\sqrt{3}}$ in the form $a + b\sqrt{3}$, where a and b are integers.

(2)
(Total 4 marks)

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15. (a) Write $\sqrt{45}$ in the form $a\sqrt{5}$, where is an integer.

(1)

(b) Express $\frac{2(3+\sqrt{5})}{(3-\sqrt{5})}$ in the form $b + c\sqrt{5}$, where b and c are integers.

(5)
(Total 6 marks)

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16. (a) Write down the value of $8^{\frac{1}{3}}$.

(1)

(b) Find the value of $8^{-\frac{2}{3}}$.

(2)
(Total 3 marks)

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17. (a) Write down the value of $16^{\frac{1}{2}}$.

(1)

(b) Find the value of $16^{-\frac{3}{2}}$.

(2)
(Total 3 marks)

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18. Given that

$$f(x) = x^2 - 6x + 18, \quad x \geq 0,$$

(a) express f(x) in the form $(x - a)^2 + b$, where a and b are integers.

(3)

The curve C with equation $y = f(x)$, $x \geq 0$, meets the y-axis at P and has a minimum point at Q.

(b) Sketch the graph of C, showing the coordinates of P and Q.

(4)

The line $y = 41$ meets C at the point R.

(c) Find the x-coordinate of R, giving your answer in the form $p + q\sqrt{2}$, where p and q are integers.

(5)
(Total 12 marks)

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19. Solve the equation $2^{1-x} = 4^x$.

(Total 3 marks)

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20. Giving your answers in the form $a + b\sqrt{2}$, where a and b are rational numbers, find

(a) $(3 - \sqrt{8})^2$,

(3)

(b) $\dfrac{1}{4-\sqrt{8}}$.

(3)
(Total 6 marks)

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21. The curve C with equation $y = \mathrm{f}(x)$ is such that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 3\sqrt{x} + \frac{12}{\sqrt{x}}, \ x > 0.$$

(a) Show that, when $x = 8$, the exact value of $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ is $9\sqrt{2}$.

(3)

The curve C passes through the point (4, 30).

(b) Using integration, find $\mathrm{f}(x)$.

(6)
(Total 9 marks)

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22. (a) Given that $3^x = 9^{y-1}$, show that $x = 2y - 2$.

(2)

(b) Solve the simultaneous equations

$$x = 2y - 2,$$

$$x^2 = y^2 + 7.$$

(6)
(Total 8 marks)

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23. Express

$$\frac{2\sqrt{2}}{\sqrt{3}-1} - \frac{2\sqrt{3}}{\sqrt{2}+1},$$

in the form $p\sqrt{6} + q\sqrt{3} + r\sqrt{2}$, where the integers p, q and r are to be found.

(Total 4 marks)

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24. Find the value of

(a) $81^{\frac{1}{2}}$,

(1)

(b) $81^{\frac{3}{4}}$,

(2)

(c) $81^{-\frac{3}{4}}$.

(1)
(Total 4 marks)

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1. $\left(\sqrt{75}-\sqrt{27}\right)=5\sqrt{3}-3\sqrt{3}$ M1

$= 2\sqrt{3}$ A1

Note

M1 for $5\sqrt{3}$ from $\sqrt{75}$ or $3\sqrt{3}$ from $\sqrt{27}$ seen anywhere
or $k = 2$, $x = 3$

A1 for $2\sqrt{3}$; allow $\sqrt{12}$ or
allow $k = 1$, $x = 12$

Some Common errors

$\sqrt{75} - \sqrt{27} = \sqrt{48}$ leading to $4\sqrt{3}$ is M0A0

$25\sqrt{3} - 9\sqrt{3} = 16\sqrt{3}$ is M0A0

[2]

2. (a) $(7 + \sqrt{5})(3 - \sqrt{5}) = 21 - 5 + 3\sqrt{5} - 7\sqrt{5}$ Expand to get 3 or 4 terms M1

$= 16, - 4\sqrt{5}$ (1st A for 16, 2nd A for $-4\sqrt{5}$) A1 A1 3

(i.s.w. if necessary, e.g. $16 - 4\sqrt{5} \rightarrow 4 - \sqrt{5}$)

Note

M1: Allowed for an attempt giving 3 or 4 terms, with at least 2 correct (even if unsimplified).

e.g. $21 - \sqrt{5}^2 + \sqrt{15}$ scores M1.

Answer only: $16 - 4\sqrt{5}$ scores full marks

One term correct scores the M mark by implication,

e.g. $26 - 4\sqrt{5}$ scores M1 A0 A1

(b) $\frac{7+\sqrt{5}}{3+\sqrt{5}} \times \frac{3-\sqrt{5}}{3-\sqrt{5}}$ (This is sufficient for the M mark) M1

Correct denominator without surds, i.e. $9 - 5$ or 4 A1

$4 - \sqrt{5}$ or $4 - 1\sqrt{5}$ A1 3

Note

Answer only: $4 - \sqrt{5}$ scores full marks

One term correct scores the M mark by implication,

e.g. $4 + \sqrt{5}$ scores M1 A0 A0

$16 - \sqrt{5}$ scores M1 A0 A0

Ignore subsequent working, e.g. $4 - \sqrt{5}$ so $a = 4$, $b = 1$

Note that, as always, A marks are dependent upon the preceding M mark, so that, for example, $\frac{7+\sqrt{5}}{3+\sqrt{5}} \times \frac{3+\sqrt{5}}{3-\sqrt{5}} = \frac{\dots\dots\dots}{4}$ is M0 A0.

Alternative

$(a + b\sqrt{5})(3 + \sqrt{5}) = 7 + \sqrt{5}$, then form simultaneous equations in a and b. M1

Correct equations: $3a + 5b = 7$ and $3b + a = 1$ A1

$a = 4$ and $b = -1$ A1

[6]

3. (a) $\left(3\sqrt{7}\right)^2 = 63$ B1 1

Note

B1 for 63 only

(b) $\left(8+\sqrt{5}\right)\left(2-\sqrt{5}\right) = 16 - 5 + 2\sqrt{5} - 8\sqrt{5}$ M1

$= 11, -\ 6\sqrt{5}$ A1, A1 3

Note

M1 for an attempt to expand <u>their</u> brackets with $\geq$ 3 terms correct.

They may collect the $\sqrt{5}$ terms to get $16 - 5 -\ 6\sqrt{5}$

Allow $-\ \sqrt{5}\times\sqrt{5}$ or $-\left(\sqrt{5}\right)^2$ or $-\sqrt{25}$ instead of the - 5

These 4 values may appear in a list or table but they should have minus signs included

The next two marks should be awarded for the final answer but check that correct values follow from correct working. Do not use ISW rule

1st A1 for 11 from 16 – 5 <u>or</u> $-6\sqrt{5}$ from $-8\sqrt{5}+2\sqrt{5}$

2nd A1 for **both** 11 and $-6\sqrt{5}$.

<u>S.C – Double sign error in expansion</u>

For $16 - 5 -\ 2\sqrt{5}\ +\ 8\sqrt{5}$ leading to 11 + … allow <u>one</u> mark

[4]

4. $32 = 2^5$ or $2048 = 2^{11}$, $\sqrt{2} = 2^{\frac{1}{2}}$ or $\sqrt{2048} = (2048)^{\frac{1}{2}}$ B1, B1

$a = \frac{11}{2}$ $\left(\text{or } 5\frac{1}{2} \text{ or } 5.5\right)$ B1

Note

1st B1 for $32 = 2^5$ or $2048 = 2^{11}$

This should be explicitly seen: $32\sqrt{2} = 2^a$ followed by $2^5\sqrt{2} = 2^a$ is OK Even writing $32 \times 2 = 2^5 \times 2 (=2^6)$ is OK but simply writing $32 \times 2 = 2^6$ is NOT

2nd B1 for $2^{\frac{1}{2}}$ or $(2048)^{\frac{1}{2}}$ seen. This mark may be implied

3rd B1 for answer as written. **Need *a* = ...** so $2^{\frac{11}{2}}$ is B0

$a = \frac{11}{2}\left(\text{or } 5\frac{1}{2} \text{ or } 5.5\right)$ with no working scores full marks.

If a = 5.5 seen then award 3/3 unless it is clear that the value follows from totally incorrect work.

Part solutions: e.g. $2^5\sqrt{2}$ scores the first B1.

Special case:

If $\sqrt{2} = 2^{\frac{1}{2}}$ is not explicitly seen, but the final answer includes $\frac{1}{2}$,

e.g. $a = 2\frac{1}{2}, a = 4\frac{1}{2}$, the second B1 is given by implication.

[3]

5. (a) $\left[(3-4\sqrt{x})^2 = \right] 9-12\sqrt{x}-12\sqrt{x}+(-4)^2 x$ M1

$9x^{-\frac{1}{2}}+16x^{\frac{1}{2}}-24$ A1, A1 3

Note

M1 for an attempt to expand $(3-4\sqrt{x})^2$ with at least 3 terms correct– as printed or better

Or $9-k\sqrt{x}+16x\,(k\neq 0)$. See also the MR rule below

1st A1 for their coefficient of $\sqrt{x}$ = 16. Condone writing (±) $9x^{(\pm)\frac{1}{2}}$ instead of $9x^{-\frac{1}{2}}$

2nd A1 for B = – 24 or their constant term = – 24

(b) f'(x) = $-\frac{9}{2}x^{-\frac{3}{2}},+\frac{16}{2}x^{-\frac{1}{2}}$ M1 A1, A1ft 3

Note

M1 for an attempt to differentiate an x term $x^n \to x^{n-1}$

1st A1 for $-\frac{9}{2}x^{-\frac{3}{2}}$ <u>and</u> their constant B differentiated to zero. NB $-\frac{1}{2}\times 9x^{-\frac{3}{2}}$ is A0

2nd A1ft follow through their $Ax^{\frac{1}{2}}$ but can be scored without a value for A, i.e. for $\frac{A}{2}x^{-\frac{1}{2}}$

(c) f ′(9) = $-\frac{9}{2}\times\frac{1}{27}+\frac{16}{2}\times\frac{1}{3}=-\frac{1}{6}+\frac{16}{6}=\frac{5}{2}$ M1 A1 2

Note

M1 for some correct substitution of x = 9 in their expression for f′(x) including an attempt at $(9)^{\pm\frac{k}{2}}$ (k odd) somewhere that leads to some appropriate multiples of $\frac{1}{3}$ or 3

A1 accept $\frac{15}{6}$ or any exact equivalent of 2.5 e.g. $\frac{45}{18},\frac{135}{54}$ or even $\frac{67.5}{27}$

Misread (MR) Only allow MR of the form $\frac{(3-k\sqrt{x})^2}{\sqrt{x}}$ N.B. Leads to answer in (c) of $\frac{k^2-1}{6}$ Score as M1A0A0, M1A1A1ft, M1A1ft

[8]

6.	(a)	**5**	(± 5 is B0)	B1	1
	(b)	$\frac{1}{(\text{their } 5)^2}$ or $\left(\frac{1}{\text{their } 5}\right)^2$		M1	
		$= \frac{1}{25}$ **or 0.04**	$(\pm\frac{1}{25}$ is A0)	A1	2

Note

M1 follow through their value of 5. Must have reciprocal and square.

5^{-2} is <u>not</u> sufficient to score this mark, unless $\frac{1}{5^2}$ follows this.

A negative introduced at any stage can score the M1 but not the A1,

e.g. $125^{-2/3} = \left(-\frac{1}{5}\right)^2 = \frac{1}{25}$ scores M1 A0

$125^{-2/3} = \left(-\frac{1}{5}\right)^2 = -\frac{1}{25}$ scores M1 A0.

Correct answer with no working scores both marks.

<u>Alternative</u>: $= \frac{1}{\sqrt[3]{125^2}}$ or $\frac{1}{(125^2)^{1/3}}$ M1 (reciprocal and the correct number squared)

$$\left(= \frac{1}{\sqrt[3]{15625}}\right)$$

$= \frac{1}{25}$ A1

[3]

7. $\sqrt{7}^2 + 2\sqrt{7} - 2\sqrt{7} - 2^2$, or 7 – 4 or an exact equivalent such as $\sqrt{49} - 2^2$ M1

3 A1

Note

M1 for an expanded expression. At worst, there can be <u>one wrong term</u> and <u>one wrong sign</u>, or <u>two wrong signs</u>.

e .g. $7 + 2\sqrt{7} - 2\sqrt{7} - 2$ is M1 (one wrong term – 2)

$7 + 2\sqrt{7} + 2\sqrt{7}$ 4 is M1 (two wrong signs $+ 2\sqrt{7}$ and + 4)

$7 + 2\sqrt{7} + 2\sqrt{7} + 2\sqrt{7} + 2$ is M1 (one wrong term + 2, one wrong sign $+ 2\sqrt{7}$

$\sqrt{7} + 2\sqrt{7} - 2\sqrt{7} + 4$ is M1 (one wrong term $\sqrt{7}$, one wrong sign + 4)

$\sqrt{7} + 2\sqrt{7} - 2\sqrt{7} - 2$ is M0 (two wrong terms $\sqrt{7}$ and – 2)

$7 + \sqrt{14} - \sqrt{14} - 4$ is M0 (two wrong terms $\sqrt{14}$ and $-\sqrt{14}$

If only 2 terms are given, they must be correct, i.e. (7 – 4) or an equivalent unsimplified version to score M1.

The terms can be seen <u>separately</u> for the M1.

Correct answer with <u>no working</u> scores both marks.

[2]

8. (a) $2x^{3/2}$ or $p = \frac{3}{2}$ (Not $2x\sqrt{x}$) B1

$-x$ or $-x^1$ or $q = 1$ B1 2

Note

1^{st} B1 for p = 1.5 or exact equivalent
2^{nd} B1 for q = 1

(b) $\left(\frac{dy}{dx} =\right) 20x^3 + 2 \times \frac{3}{2} x^{1/2} - 1$ M1

$20x^3 + 3x^{\frac{1}{2}} - 1$ A1A1ftA1ft 4

Note

M1 for an attempt to differentiate $x^n \rightarrow x^{n-1}$ (for any of the 4 terms)
1^{st} A1 for $20x^3$ (the – 3 must 'disappear')

2^{nd} A1ft for $3x^{\frac{1}{2}}$ or $3\sqrt{x}$. Follow through their p but they must be differentiating $2x^p$, where p is a fraction, and the coefficient must be simplified if necessary.

3^{rd} A1ft for –1 (not the unsimplified $-x^0$), or follow through for correct differentiation of their $-x^q$ (i.e. coefficient of x^q is – 1). If ft is applied, the coefficient must be simplified if necessary.

'Simplified' coefficient means $\frac{a}{b}$ where a and b are integers with no common

factors. Only a single + or – sign is allowed (e.g. – – must be replaced by +).

If there is a 'restart' in part (b) it can be marked independently of part (a), but marks for part (a) cannot be scored for work seen in (b).

Multiplying by $\sqrt{x}$: (assuming this is a restart)

e.g. $y = 5x^4\sqrt{x} - 3\sqrt{x} + 2x^2 - x^{3/2}$

$\left(\frac{dy}{dx} =\right) \frac{45}{2} x^{7/2} - \frac{3}{2} x^{-1/2} + 4x - \frac{3}{2} x^{1/2}$ scores M1 A0 A0 (p not a fraction) A1ft.

Extra term included: This invalidates the final mark.

e.g. $y = 5x^4 - 3 + 2x^2 - x^{3/2} - x^{1/2}$

$\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)20x^3+4x-\frac{3}{2}x^{1/2}-\frac{1}{2}x^{-1/2}$ scores M1 A1 A0 (p not a fraction) A0.

Numerator and denominator differentiated separately:
For this, neither of the last two (ft) marks should be awarded.

Quotient/product rule:
Last two terms must be correct to score the last 2 marks. (If the M mark has not already been earned, it can be given for the quotient/product rule attempt.)

[6]

9.	(a)	2	B1	1
		<u>Negative answers:</u>		
		Allow –2. Allow ±2. Allow '2 or –2'.		
	(b)	x^9 seen, or (answer to (a))^3 seen, or $(2x^3)^3$ seen.	M1	
		$8x^9$	A1	2
		M: Look for x^9 first… if seen, this is M1.		
		If not seen, look for (answer to (a))^3, e.g. 2^3… this would score M1 even if it does not subsequently become 8. (Similarly for other answers to (a)).		
		In $(2x^3)^3$, the 2^3 is implied, so this scores the M mark.		
		<u>Negative answers:</u>		
		Allow $\pm 8x^9$. Allow '$8x^9$ or $-8x^9$'.		

N.B. If part (a) is wrong, it is possible to 'restart' in part (b) and to score full marks in part (b).

[3]

10. $\dfrac{(5-\sqrt{3})}{(2+\sqrt{3})}\times\dfrac{(2-\sqrt{3})}{(2-\sqrt{3})}$ M1

$= \dfrac{10-2\sqrt{3}-5\sqrt{3}+(\sqrt{3})^2}{\dots} \quad \left(= \dfrac{10-7\sqrt{3}+3}{\dots}\right)$ M1

$(=13-7\sqrt{3}) \quad \left(\text{Allow } \dfrac{13-7\sqrt{3}}{1}\right)$ 13 (a = 13) A1

$-7\sqrt{3}$ ($b = -7$) A1 4

1st M: Multiplying top and bottom by (2 – $\sqrt{3}$). (As shown above is sufficient).

2nd M: Attempt to multiply out numerator (5 – $\sqrt{3}$) (2 – $\sqrt{3}$). Must have at least 3 terms correct.

Final answer: Although 'denominator = 1' may be implied, the $13-7\sqrt{3}$ must obviously be the final answer (not an intermediate step), to score full marks. (Also M0 M1 A1 A1 is not an option).

The A marks cannot be scored unless the 1st M mark has been scored, but this 1st M mark could be implied by correct expansions of both numerator and denominator.

It is possible to score M1 M0 A1 A0 or M1 M0 A0 A1 (after 2 correct terms in the numerator).

Special case: If numerator is multiplied by $(2+\sqrt{3})$ instead of $(2-\sqrt{3})$, the 2nd M can still be scored for at least 3 of these terms correct: $10-2\sqrt{3}+5\sqrt{3}-(\sqrt{3})^2$.
The maximum score in the special case is 1 mark: M0 M1 A0 A0.

Answer only: Scores no marks.

Alternative method:

$5-\sqrt{3}=(a+b\sqrt{3})(2+\sqrt{3})$

$(a+b\sqrt{3})(2+\sqrt{3}) = 2a+a\sqrt{3}+2b\sqrt{3}+3$ M1: At least 3 terms correct.

$5 = 2a + 3b$

$-1 = a + 2b$ a =… or b =… M1: Form and attempt to solve simultaneous equations.

a = 13, b = –7 A1, A1

[4]

11. $9 - 5$ or $3^2 + 3\sqrt{5} - 3\sqrt{5} - \sqrt{5} \times \sqrt{5}$ or $3^2 - \sqrt{5} \times \sqrt{5}$ or $3^2 - (\sqrt{5})^2$ M1
$= \underline{4}$ A1cso 2

M1 for an attempt to multiply out. There must be at least 3 correct terms. Allow one sign slip only, no arithmetic errors.

e.g. $3^2 + 3\sqrt{5} - 3\sqrt{5} + (\sqrt{5})^2$ is M1A0

$3^2 + 3\sqrt{5} + 3\sqrt{5} - (\sqrt{5})^2$ is M1A0 as indeed is $9 \pm 6\sqrt{5} - 5$

BUT $9 + \sqrt{15} - \sqrt{15} - 5(= 4)$ is M0A0 since there is more than a sign error.
$6 + 3\sqrt{5} - 3\sqrt{5} - 5$ is M0A0 since there is an arithmetic error.

If all you see is 9 ± 5 that is M1 but please check it has not come from incorrect working.

Expansion of $(3+\sqrt{5})(3+\sqrt{5})$ is M0A0

A1cso for 4 only. Please check that no incorrect working is seen.

Correct answer only scores both marks.

[2]

12. (a) Attempt $\sqrt[3]{8}$ or $\sqrt[3]{(8^4)}$ M1

$= \underline{16}$ A1 2

M1 for: 2 (on its own) or $(2^3)^{\frac{4}{3}}$ or $\sqrt[3]{8}$ or $(\sqrt[3]{8})^4$ or 2^4

or $\sqrt[3]{8^4}$ or $\sqrt[3]{4096}$

8^3 or 512 or $(4096)^{\frac{1}{3}}$ is M0

A1 for 16 only

(b) $\underline{5x^{\frac{1}{3}}}$ $5,\ x^{\frac{1}{3}}$ B1, B1 2

1st B1 for 5 on its own or × something.

So e.g. $\dfrac{5x^{\frac{4}{3}}}{x}$ is B1 But $5^{\frac{1}{3}}$ is B0

An expression showing cancelling is not sufficient (see first expression of QC0184500123945 the mark is scored for the second expression)

2nd B1 for $x^{\frac{1}{3}}$

Can use ISW (incorrect subsequent working)

e.g. $5x^{\frac{4}{3}}$ scores B1B0 but it may lead to $\sqrt[3]{5x^4}$ which we ignore as ISW.

Correct answer only scores full marks in both parts.

[4]

13. (a) $6\sqrt{3}$ ($a = 6$) B1 1

$\pm 6\sqrt{3}$ also scores B1.

(b) Expanding $(2 - \sqrt{3})^2$ to get 3 or 4 separate terms M1

$7, -4\sqrt{3}$ ($b = 7$, $c = -4$) A1, A1 3

M1: The 3 or 4 terms may be wrong.

1st A1 for 7, 2nd A1 for $-4\sqrt{3}$.

Correct answer $7 - 4\sqrt{3}$ with no working scores all 3 marks.

$7 + 4\sqrt{3}$ with or without working scores M1 A1 A0.

Other wrong answers with no working score no marks.

[4]

14. (a) $16+4\sqrt{3}-4\sqrt{3}-(\sqrt{3})^2$ or $16-3$ M1

$= 13$ A1c.a.o 2

M1 For 4 terms, at least 3 correct

e.g. $8 + 4\sqrt{3}-4\sqrt{3}-(\sqrt{3})^2$ *or* $16\pm 8\sqrt{3}-(\sqrt{3})^2$ *or*

16 + 3

4^2 *instead of 16 is OK*

$(4+\sqrt{3})(4+\sqrt{3})$ *scores M0A0*

(b) $\frac{26}{4+\sqrt{3}}\times\frac{4-\sqrt{3}}{4-\sqrt{3}}$ M1

$=\frac{26(4-\sqrt{3})}{13}=8-2\sqrt{3}$ or $8+(-2)\sqrt{3}$ or $a = 8$ and $b = -2$ A1 2

M1 For a correct attempt to rationalise the denominator

Can be implied

NB $\frac{-4+\sqrt{3}}{-4+\sqrt{3}}$ *is OK*

[4]

15. (a) $3\sqrt{5}$ (or $a = 3$) B1 1

(b) $\frac{2(3+\sqrt{5})}{(3-\sqrt{5})}\times\frac{(3+\sqrt{5})}{(3+\sqrt{5})}$ M1

$(3-\sqrt{5})(3+\sqrt{5}) = 9-5$ (= 4) (Used as or intended as denominator) B1

$(3+\sqrt{5})(p \pm q\sqrt{5}) = \ldots$ 4 terms ($p \neq 0$, $q \neq 0$) (Independent) M1
or $(6+\sqrt{5})(p \pm q\sqrt{5}) = \ldots$ 4 terms ($p \neq 0$, $q \neq 0$)

[Correct version: $(3+\sqrt{5})(3+\sqrt{5}) = 9 + 3\sqrt{5} + 3\sqrt{5} + 5$,or double this.]

$\frac{2(14+6\sqrt{5})}{4} = 7+3\sqrt{5}$ 1st A1: $b = 7$, 2nd A1: $c = 3$ A1 A1 5

[6]

2nd M mark for attempting $(3+\sqrt{5})(p+q\sqrt{5})$ is generous. Condone errors.

16. (a) $\underline{2}$ B1 1

Penalise $\pm$

(b) $8^{-\frac{2}{3}} = \frac{1}{\sqrt[3]{64}}$ or $\frac{1}{(a)^2}$ or $\frac{1}{\sqrt[3]{8^2}}$ or $\frac{1}{8^{\frac{2}{3}}}$ M1

Allow $\pm$

$= \frac{1}{4}$ or 0.25 A1 2

M1 for understanding that "-" power means reciprocal

$8^{\frac{2}{3}} = 4$ *is M0A0 and* $-\frac{1}{4}$ *is M1A0*

[3]

17. (a) 4 (or $\pm$ 4) B1

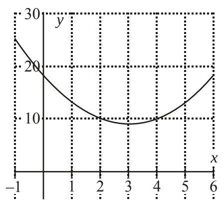
(b) $16^{-\frac{3}{2}} = \frac{1}{16^{\frac{3}{2}}}$ and any attempt to find $16^{\frac{3}{2}}$ M1

$\frac{1}{64}$ (or exact equivalent, e.g. 0.015625) (or $\pm\frac{1}{64}$) A1 3

[3]

18. (a) $x^2 - 6x + 18 = (x - 3)^2, +9$ B1, M1 A1 3

(b)



"U"-shaped parabola M1
Vertex in correct quadrant A1ft
P: (0, 18) (or 18 on y-axis) B1
Q: (3, 9) B1ft 4

(c) $x^2 - 6x + 18 = 41$ or $(x - 3)^2 + 9 = 41$ M1
Attempt to solve 3 term quadratic $x = \ldots$ M1

$x = \frac{6 \pm \sqrt{36 - (4 \times -23)}}{2}$ (or equiv.) A1

$\sqrt{128} = \sqrt{64} \times \sqrt{2}$ (or surd manipulation $\sqrt{2a} = \sqrt{2}\sqrt{a}$)M1

$3 + 4\sqrt{2}$ A1 5

[12]

19.	$4 = 2^2$ (or log 4 = 2log 2)		B1	
	Linear equation in x: $1 - x = 2x$		M1	
	$x = \frac{1}{3}$		A1	3

[3]

20.	(a)	$\sqrt{8} = \sqrt{4}\sqrt{2} = 2\sqrt{2}$ (seen or implied)	B1	
		$(3 - \sqrt{8})(3 - \sqrt{8}) = 9 - 6\sqrt{8} + 8 = 17 - 12\sqrt{2}$	M1 A1	3
	(b)	$\frac{1}{4-\sqrt{8}} \times \frac{4+\sqrt{8}}{4+\sqrt{8}}, \quad = \frac{4+\sqrt{8}}{16-8} = \frac{1}{2} + \frac{1}{4}\sqrt{2}$	M1, M1 A1	3
		Allow $\frac{1}{4}(2+\sqrt{2})$ *or equiv. (in terms of* $\sqrt{2}$*)*		

[6]

21.	(a)	$\sqrt{8} = 2\sqrt{2}$ seen or used somewhere (possibly implied).	B1	
		$\frac{12}{\sqrt{8}} = \frac{12\sqrt{8}}{8}$ or $\frac{12}{2\sqrt{2}} = \frac{12\sqrt{2}}{4}$		
		Direct statement, e.g. $\frac{6}{\sqrt{2}} = 3\sqrt{2}$ (no indication of method) is M0.	M1	
		At $x = 8$, $\frac{dy}{dx} = 3\sqrt{8} + \frac{12}{\sqrt{8}} = 6\sqrt{2} + 3\sqrt{2} = 9\sqrt{2}$ (*)	A1	3
	(b)	Integrating: $\frac{3x^{3/2}}{\left(3/2\right)} + \frac{12x^{1/2}}{\left(1/2\right)} (+C)$ (C not required)	M1 A1 A1	
		At (4, 30), $\frac{3 \times 4^{3/2}}{\left(3/2\right)} + \frac{12 \times 4^{1/2}}{\left(1/2\right)} + C = 30$ (C required)	M1	
		(f(x) =) $2x^{3/2} + 24x^{1/2}$, -34	A1, A1	6

[9]

22.	(a)	$3^x = 3^{2(y-1)}$ $\quad x = 2\,(y-1)$ (*)	M1 A1	
	(b)	$(2y-2)^2 = y^2 + 7, \quad 3y^2 - 8y - 3 = 0$	M1, A1	
		$(3y+1)(y-3) = 0$, $y = \ldots$ (or correct substitution in formula)	M1	
		$y = -\frac{1}{3}, \quad y = 3$	A1	
		$x = -\frac{8}{3}, \quad x = 4$	M1 A1 ft	

[8]

23.	Rationalising either surd	M1	
	$\frac{2\sqrt{2}(\sqrt{3}+1)}{2} - \frac{2\sqrt{3}(\sqrt{2}-1)}{1}$	A1, A1	
	$-\sqrt{6} + 2\sqrt{3} + \sqrt{2}$ $\quad$ (or $p = -1$, $q = 2$, $r = 1$)	A1	4

[4]

24.	(a)	9		B1	1
	(b)	$81^{\frac{1}{4}} = 3$	$3^3 = 27$	M1 A1	2
	(c)	$\frac{1}{27}$		B1 ft	1

[4]

1. This was a successful starter to the paper with very few candidates failing to attempt it and many securing both marks. The $\sqrt{27}$ was usually written as $3\sqrt{3}$ but $5\sqrt{5}$ and $3\sqrt{5}$ were common errors for $\sqrt{75}$. The most common error though was to subtract 27 from 75 and then try and simplify $\sqrt{48}$ which showed a disappointing lack of understanding.

2. This question was answered very well, with many candidates scoring full marks. Mistakes in part (a) were usually from incorrect squaring of the $\sqrt{5}$ term, sign errors or errors in collecting the terms. In part (b), the method for rationalising the denominator was well known and most candidates, whether using their answer to part (a) or not, proceeded to a solution. A common mistake, however, was to divide only one of the terms in the numerator by 4.

3. Some candidates could not square the surd terms correctly but nearly everyone attempted this question and most scored something.

In part (a) some failed to square the 3 and an answer of 21 was fairly common, others realised that the expression equalled 9 × 7but then gave the answer as 56. A few misread the question and proceeded to expand $(3+\sqrt{7})^2$. In part (b) most scored a mark for attempting to expand the brackets but some struggled here occasionally adding 8 + 2 instead of multiplying. Those with a correct expansion sometimes lost marks for careless errors, $-8\sqrt{5}+2\sqrt{5}+=6\sqrt{5},$ and a small number showed how fragile their understanding of these mathematical quantities was by falsely simplifying a correct answer of $11-6\sqrt{5}$ to $5\sqrt{5}$.

4. Many candidates could not deal with this test of indices. Two simple properties of indices were required: that a square root leads to a power of $\frac{1}{2}$ and the rule for adding the powers when multiplying. Those who identified these usually made good progress but the remainder struggled. Some re-wrote $32\sqrt{2}$ as $\sqrt{64}$ and then obtained a = 3 whilst others did obtain $\sqrt{2048}$ but usually failed to identify 2048 as 2^{11}. A number of candidates tried to use logs but this approach was rarely successful.

5. Most candidates made a good attempt at expanding the brackets but some struggled with $-4\sqrt{x}\times -4\sqrt{x}$ with answers such as $-4\sqrt{x}$ or $\pm 16\sqrt{x}$ or $\pm 16x^{\frac{1}{4}}$ being quite common. The next challenge was the division by $\sqrt{x}$ and some thought that $\frac{x}{\sqrt{x}}=1$. Many, but not all, who had difficulties in establishing the first part made use of the given expression and there were plenty of good attempts at differentiating. Inevitably some did not interpret f '(x) correctly and a few attempted to integrate but with a follow through mark here many scored all 3 marks. In part (c) the candidates were expected to evaluate $9^{-\frac{3}{2}}$ or $9^{-\frac{1}{2}}$ correctly and then combine the fractions - two significant challenges but many completed both tasks very efficiently.

6. Many candidates answered both parts of this question correctly. In part (b), however, some did not understand the significance of the negative power. Others, rather than using the answer to part (a), gave themselves the difficult, time-wasting task of squaring the 125 and then attempting to find a cube root. Negative answers (or ±) appeared occasionally in each part of the question.

7. Most candidates completed this question successfully, either by expanding the brackets to find four terms or by recognising the difference of squares and writing down 7 – 4 = 3 directly. Common wrong answers included $11 + 4\sqrt{7}$, from $(\sqrt{7}+2)(\sqrt{7}+2)$, and 5, from $7 + 2\sqrt{7} - 2\sqrt{7} - 2$. Mistakes such as $\sqrt{7}\times 2 = \sqrt{14}$ were rarely seen.

8. Good candidates generally had no difficulty with the division in part (a) of this question, but others were often unable to cope with the required algebra and produced some very confused solutions. A common mistake was to "multiply instead of divide", giving $2x^2 \div \sqrt{x} = 2x^{\frac{5}{2}}$, and sometimes $\sqrt{x}$ was interpreted as x^{-1}. Examiners saw a wide variety of wrong answers for p and q.
Most candidates were able to pick up at least two marks in part (b), where follow-through credit was available in many cases. While the vast majority used the answers from part (a), a few differentiated the numerator and denominator of the fraction term separately, then divided.

9. Although many candidates obtained the correct answer in part (a), they were usually unable to use this to find $(16x^{12})^{\frac{3}{4}}$ in part (b). Even very good candidates tended to score only one mark here, with the most common incorrect answers being $16x^9$ and $12x^9$. Other wrong answers included those in which powers had been added to give $x^{\frac{51}{4}}$. Very confused algebra was sometimes seen.

10. There were many completely correct solutions to this question. The majority of candidates knew the correct method of multiplying the numerator and denominator by $(2 - \sqrt{3})$ and many were correct in the arithmetic manipulation. Some multiplied incorrectly by $(2 + \sqrt{3})$ or by $(5 + \sqrt{3})$. A number of candidates were unable to square $\sqrt{3}$ correctly and it was disappointing to see marks lost though careless arithmetic. Only a small minority of candidates had no idea of how to start.

11. This proved an easy starter for most candidates. Some identified this as a difference of two squares, and simply wrote down 9 – 5, but most opted to multiply out and sign slips spoilt some answers with 9+5 appearing quite often. Others thought that $3\sqrt{5} = \sqrt{15}$ and some wrote $3^2 = 6$.

12. There were many correct responses to both parts of this question. In part (a) some reached $\sqrt[3]{4096}$ but could not simplify this expression but most managed $\sqrt[3]{8} = 2$ and usually went on to give the final answer of 16. A few attempted $\left(\sqrt[4]{8}\right)^3$ but most interpreted the notation correctly. Part (b) revealed a variety of responses from those whose grasp of the basic rules of algebra is poor. Most simplified the numerical term to 5 but often they seemed to think the *x* terms "disappeared" and answers of $5^{\frac{1}{3}}$ or $5^{\frac{4}{3}}$ were common. Dealing with the *x* term proved quite a challenge for some and $(5x)^{\frac{4}{3}}$ was a common error. Some candidates tried to "simplify" a correct answer, replacing $5x^{\frac{1}{3}}$ with $\sqrt[3]{5x}$. On this occasion the examiners ignored this subsequent working but such a misunderstanding of the mathematical notation used in AS level mathematics is a legitimate area to be tested in future.

13. It was encouraging to see most candidates factorizing the quadratic expression in order to find the critical values for the inequality. Sometimes the critical values had incorrect signs, despite the factorization being correct, but the most common error was still a failure to select the outside region. Some candidates struggled with the correct symbolic notation for the answer and $-2 > x > 9$ was occasionally seen.
A few candidates chose to use the formula or completing the square to find the critical values, these approaches are less efficient in this case and often gave rise to arithmetic errors.

14. This question was generally answered well although there was the usual crop of arithmetic and sign errors especially in part (a) where some candidates struggled to simplify $\left(\sqrt{3}\right)^2$. In part (b) most knew how to start the problem, although a few multiplied by $\frac{4+\sqrt{3}}{4+\sqrt{3}}$. It was disappointing to see how many candidates multiplied out the numerator first and then divided by 13, often forgetting to divide one of the two terms by 13.

15. The surd simplification in part (a) was completed successfully by most candidates, although 9√5 was sometimes seen instead of 3√5. In part (b), candidates often showed competence in the process of rationalising the denominator, but sometimes made mistakes in multiplying out their brackets, with frequent mishandling of terms involving √5. It was disappointing to see a few candidates proceeding correctly to $\frac{28+12\sqrt{5}}{4}$ and then dividing only one of the terms, to give, for example, 7 + 12√5. Not surprisingly, those who were unfamiliar with rationalising the denominator usually scored no marks in part (b).

16. This was a successful starter to the paper and nearly all the candidates were able to make some progress. Part (a) was rarely incorrect although sometimes the answer was given as ±2. The negative power led to a number of errors in part (b). Some thought that $8^{-\frac{2}{3}} = 8^{\frac{3}{2}}$ whilst others thought that a negative answer (usually – 4) was appropriate. Most knew that the negative power meant a reciprocal and the correct answer was often seen.

17. There were many completely correct answers to this question. The vast majority of candidates knew that a square root was required in part (a), but part (b) caused a few more difficulties, with the negative power not always being interpreted to mean a reciprocal. Some cubed 16 before finding a square root, making unnecessary work for themselves, while others evaluated $16^{\frac{3}{2}}$ as 12 or 48.

18. Most candidates were familiar with the method of "completing the square" and were able to produce a correct solution to part (a). Sketches in part (b), however, were often disappointing. Although most candidates knew that a parabola was required, the minimum point was often in the wrong position, sometimes in the fourth quadrant and sometimes at (3, 0). Some candidates omitted part (c), but most knew what was required and some very good, concise solutions were seen. Those who used the answer to part (a) and formed the equation $(x-3)^2+9=41$ were able to proceed more easily to an answer from $x = 3 \pm \sqrt{32}$. Some candidates found difficulty in manipulating surds and could not cope with the final step of expressing √32 as 4√2.

19. Most candidates completed this question successfully, although marks were sometimes lost through careless algebra. The most popular method was to write 4^x as $(2^2)^x$, then to equate powers and solve the resulting linear equation. Other approaches, including the use of logarithms, were occasionally seen.

20. Although good candidates often produced fully correct solutions to this question, others lacked confidence in using surds and found the manipulation difficult. In both parts of the question, a significant number of candidates left their answer in terms of $\sqrt{8}$, failing to demonstrate the simplification to $2\sqrt{2}$. In part (a), most earned a method mark for their attempt to square the bracket, but numerical and sign errors were common. In part (b), however, many candidates did not realise that they needed to (or did not know how to) rationalise the denominator, and therefore made little progress. It was disappointing to see attempts starting with $\frac{1}{4-\sqrt{8}} = \frac{1}{4} - \frac{1}{\sqrt{8}}$.

21. The manipulation of surds in part (a) was often disappointing in this question. While most candidates appreciated the significance of the "exact value" demand and were not tempted to use decimals from their calculators, the inability to rationalise the denominator was a common problem. Various alternative methods were seen, but for all of these, since the answer was given, it was necessary to show the relevant steps in the working to obtain full marks.

Integration techniques in part (b) were usually correct, despite some problems with fractional indices, but the lack of an integration constant limited candidates to 3 marks out of 6. Sometimes the (4, 30) coordinates were used as limits for an attempted "definite integration".

22. In part (a), most candidates were able to use $9 = 3$ to show convincingly that $x = 2y - 2$. Just a few used logarithms, usually correctly, to complete the proof. Part (b), however, proved to be a good test of algebraic competence. Forming an appropriate equation in y was the important step, and candidates who thought that $x = 2y - 2$ implied $x = 4y - 4$ (or similar) oversimplified the y equation to a two-term quadratic and limited themselves to a maximum of 2 marks.

Otherwise, apart from slips in expanding $(2y - 2)$, many went on to achieve a fully correct solution. It was unfortunate, however, that some candidates solved the quadratic in y but gave their answers as values of x, substituting back to find "y". Others, having solved for y, omitted to find the corresponding values of x.

23. No Report available for this question.

24. No Report available for this question.